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Measurement of Water Movement through Soil

–Experiments in Water Velocity Measurement in Porous Media–

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Abstract

Experiments are described for measurement of water velocity in porous media using sinusoidal heat conduction. The theoretical considerations in two earlier papers (Tominaga, 1976a,b) are summarized first. In these papers, two methods for soil-water velocity measurement were developed and the mechanisms of heat conduction in porous media were discussed. In this paper, these results are combined for measurement in porous media, and experiments are conducted on the basis of these theoretical conclusions. The water velocities in these experiments are about 0.09 – 1.56mm/s, which are true velocities in the spaces among soil particles. Glass beads are used as porous media in these experiments. The results correspond with the theoretical considerations well. Several strong points of the phase-difference method are discussed based on these results.

1. Preface

In two earlier papers (Tominaga, 1976a,b), the theoretical considerations for measurement of water velocity in porous media were developed. The amplitude method and the phase-difference method were proposed from the solution of the heat conduction of fluids, when a sinusoidal heat signal is induced. Then, the principle of heat conduction in the porous media was developed for application of these methods to the velocity of water flow in porous media. In the following sections, these theoretical considerations are summarized, and experiments on water velocity measurement by the phase-difference method in porous media are described.

Nomenclature

θ	K	temperature
u	K/s	temperature input
a	kg/m^3	density
c	J/(kg·K)	specific heat
λ	J/(m·s·K)	thermal conductivity
$K=\lambda/ac$		
	m^2/s	thermal diffusivity
t	s	time
$\boldsymbol{v}$	m/s	velocity

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N_i		ratio of S_i to S, simultaneously, V_i to V *
suffix $i = \begin{cases} f \\ s \end{cases}$		for flowing medium for stationary medium
$\overline{K}$	m^2/s	apparent thermal diffusivity in porous media
H		heat capacity ratio of flowing medium to the mixed media
R	m	distance from the origin of axis to the measuring point (in the amplitude method)
d	m	interval between the measuring points (in the phase-difference method)
ω	rad/s	angular frequency of input temperature
ϕ	rad	phase-difference
* S	m^2	area of closed surface S
S_i	m^2	area occupied by material i on closed surface S
V	m^3	volume of closed surface S
V_i	m^3	volume occupied by material i in S

2. Summary of theoretical considerations

2.1 Energy equations

Two methods presented in the first paper (Tominaga, 1976a) are obtained from an energy equation of fluid flow in which there are no obstructions to the flow. The energy equation in this case is as follows:

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K \nabla \theta) - \nabla \cdot (\theta \boldsymbol{v}) + u \qquad (1)$$

In porous media, water flow is obstructed by the porous structure and heat conduction through it adds another influence to the conduction of water itself. The resulting formulation in the second paper (Tominaga, 1976b) is as follows:

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (\overline{K} \nabla \theta) - \nabla \cdot (H \theta \boldsymbol{v}) + u \qquad (2)$$

where:

$$\overline{K} = \frac{\overline{\lambda}}{\overline{ac}}$$

$$H = \frac{a_f c_f N_f}{\overline{ac}}$$

$$\overline{\lambda} = \lambda_f N_f + \lambda_s N_s$$

$$\overline{ac} = a_f c_f N_f + a_s c_s N_s$$

The ratios of volume $m_i = V_i/V$ and that of area $n_i = S_i/S$ in the second paper are proved to be equal and are expressed by N_i in this paper. This was shown in the paper (Tominaga, 1980a).

2.2 Methods for velocity measurement in porous media

Considering the result of the second paper, the mechanism of heat conduction in porous media resembles that of the first paper under constant water velocity. Therefore, the two methods in the first paper can be rewritten for measurement in porous media.

2.2.1 Amplitude method

$$\boldsymbol{v} = \frac{\overline{K}}{RH}(\boldsymbol{i}A+\boldsymbol{j}B+\boldsymbol{k}C) \tag{3}$$

where:

$$A=\ln\frac{\theta_{+X}}{\theta_{-X}},\quad B=\ln\frac{\theta_{+Y}}{\theta_{-Y}},\quad C=\ln\frac{\theta_{+Z}}{\theta_{-Z}}$$

θ_{+X}, θ_{-X} are the amplitudes of the sinusoidal heat response measured at points $+R$, $-R$ on the X coordinate, when point heat source is set at the origin.

θ_{+Y}, θ_{-Y} and θ_{+Z}, θ_{-Z} are obtained in the same way.

2.2.2 Phase-difference method

$$\| \boldsymbol{v} \|^2 = \frac{1}{H^2}\left(\frac{d^2}{\phi^2}\ \omega^2 - 4\,\overline{K}^2\frac{\phi^2}{d^2}\right) \tag{4}$$

If two phase-differences ϕ_1 and ϕ_2 are respectively known in response to the two different angular frequencies ω_1 and ω_2 on input in regard to the same velocity, then the thermal diffusivity can be eliminated.

$$\| \boldsymbol{v} \|^2 = \frac{d^2}{H^2}\left(\frac{\omega_1^2}{\phi_1^2}+\frac{\omega_2^2}{\phi_2^2}-\frac{\omega_1^2-\omega_2^2}{\phi_1^2-\phi_2^2}\right) \tag{5}$$

$$\overline{K}^2 = \frac{d^2}{4}\ \frac{1}{\phi_1^2-\phi_2^2}\left(\frac{\omega_1^2}{\phi_1^2}-\frac{\omega_2^2}{\phi_2^2}\right) \tag{6}$$

3. Experiments with the phase-difference method

It is very difficult to know the thermal diffusivity of soil after the burial of the sensoring elements in practical applications. Therefore, the phase-difference method is more valid when the direction of flow is known in advance. In this section, experiments with the phase-difference method are presented.

3.1 Model of the porous media

As a model for the porous media, fine glass beads about 0.2mm in diameter were used. They were packed in a transparent acrylite pipe, which is shown in **Photo 1-a**, The spaces between particles were filled with water in order to regard only the glass beads as the stationary medium. In this case, the water was the only flowing medium.

N_f and N_S of the experimental pipe were as follows:

$$N_f = 0.360$$
$$N_s = 0.640$$

Other parameters of these experiments were about as follows:

$$\overline{\lambda} = 0.851\ \ \mathrm{J/(m\cdot s\cdot K)}$$
$$\overline{ac} = 2.97\times10^6\ \ \mathrm{J/(m^3\cdot K)}$$
$$\overline{K} = \overline{\lambda}\,/\,\overline{ac} = 2.87\times10^{-7}\ \ (\mathrm{m^2/s})$$
$$H = 0.508$$

These were obtained from a published data table (Chronological Table of Science, 1979).

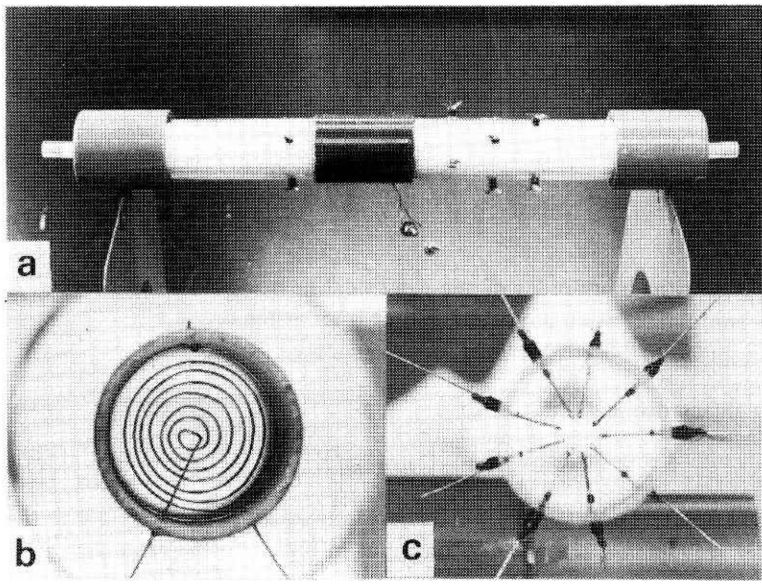

Photo 1. The experimental pipe. **(a)** Outside view. **(b)** Heater. **(c)** CC-thermocouples.

Taking the axis line of the experimental pipe as one-dimentional coordinate, spiral Nicrome wire which was 5 Ω was used as the point heat source (shown in **Photo 1-b).** Three sets of C-C (Copper-Constantan) thermocouples were used as the detectors of heat response (shown in **Photo 1-c).** Their outputs were totaled together to get large output signals. Four measuring points were set along the axis line, one upstream from the point heat source and the others in downstream (see **Fig. 1**).

3.2 Consideration of the spatial interval between measuring points

When the heat input is applied to any material, spatial frequency appears in that material.

$$g = \frac{1}{4\sqrt{2}\pi K}\sqrt{\sqrt{\| H\boldsymbol{v} \|^4 + 16K^2\omega^2} - \| H\boldsymbol{v} \|^2} \qquad (7)$$

where:

g = spatial frequency [1/m]

According to the sampling theorem, the spatial interval must be restricted by the following equation:

$$d \leq \frac{1}{2g_c} \qquad (8)$$

where:

g_c = spatial frequency corresponding to the cut-off frequency of input f_c

When the measuring points are set at fixed locations, f_c is determined by the following equation, inversely.

$$f_c \leq \sqrt{\frac{\pi^2 K^2}{d^4} + \frac{\| H\boldsymbol{v} \|^2}{4d^2}} \qquad (9)$$

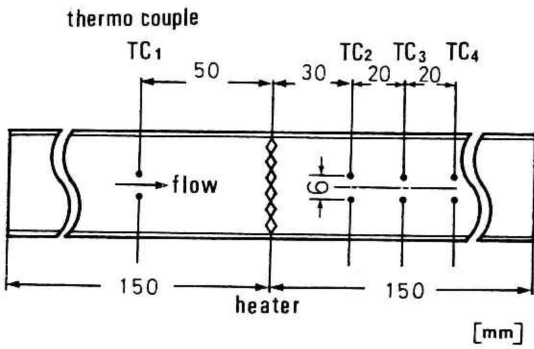

Fig. 1 The experimental pipe for measuring water velocity in porous media and locations of CC-thermocouples.

This relation assures that the phase shift in the frequency concerned is not larger than π between the two measuring points. In these experiments, the frequencies were chosen according to **Eq. 9.** The phase-differences were obtained by using the measured values of heat response taken at points TC_2 and TC_4.

3.3 Measurement of the sinusoidal heat response

Sinusoidal heat responses were measured to confirm the boundary conditions presented in the first paper (Tominaga, 1976a), in which the response is sinusoidal when sinusoidal heat input is induced. Four frequencies with periods of 50, 100, 200 and 500s were used for two velocities of about 0.67 and 2.44mm/s. Input and response signals were respectively transformed by Fourier's analysis formula in order to obtain the fundamental harmonies. The error ε between the true sinusoidal heat signal, which is the fundamental harmonic, and the actual heat signal was calculated by the following equation.

$$\varepsilon = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\theta_{t_i} - b \cos \omega t_i)^2 / b} \qquad (10)$$

where:

b = amplitude of the fundamental harmonic (K)
θ = actual temperature signal (K)
t_i = measuring time (s)
n = number of data
ω = angular frequencies of the fundamental harmonic (rad/s)

This gives the root mean square error deviates from the true sinusoidal heat signal. Substituting the induced signals and the measured signal at TC_2 into **Eq. 10** respectively, the errors were 0.012 – 0.031 for the input and 0.011 – 0.026 for the response(see **Table 1**). This means that the responses could be taken as the sinusoidal waves. The amplitudes of heat responses were under 5K, so that the heat responses were not disturbed by any noise except heat input. **Photo 2** shows an example of the sinusoidal heat response.

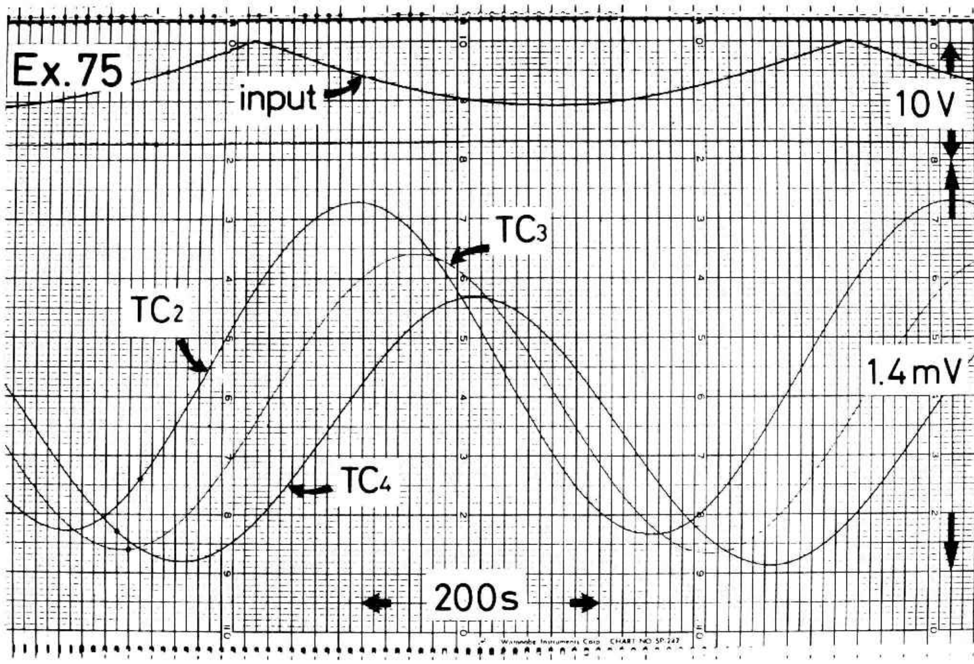

Photo 2 An example of measured heat signal in response to sinusoidal heat input.

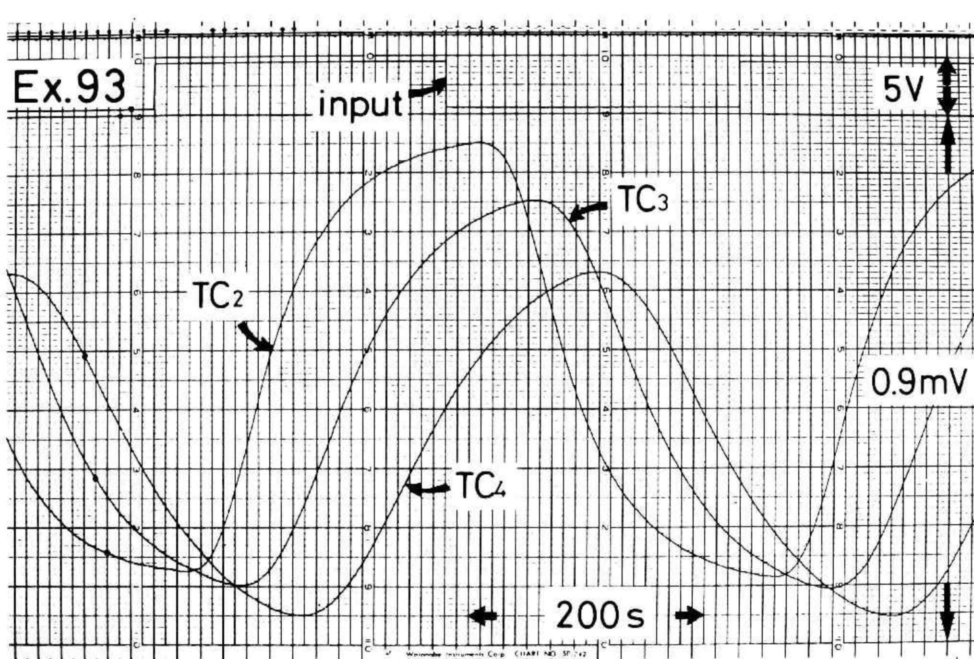

Photo 3 An example of measured heat signal in response to square wave heat input.

Table 1 Errors in sinusoidal heat response.

Exp. No.	Calibrated Vs (mm/s)	Period of Input (s)	ε of Input	ε of Output TC2
72	0.671	50.8	0.01165	0.01582
73	0.664	101.8	0.01178	0.01929
74	0.673	201.0	0.01695	0.01052
75	0.677	506.2	0.01644	0.01165
76	2.474	49.4	0.01292	0.02016
77	2.480	101.5	0.01312	0.02599
78	2.426	202.0	0.02643	0.01239
79	2.420	489.2	0.03110	0.01184

Table 2 Experimental conditions and measured values.

Exp. No.	Calibrated Vs (mm/s)	Period of Input (s)	Φ_1 (rad)	Φ_3 (rad)	Vp (mm/s) by Eq. (5)	$\overline{K}$ ($\times 10^{-7}$ m^2/s) by Eq. (6)
88	0.085	2000	2.2046	5.0002	0.102	2.199
87	0.087	1000	3.5658	8.2134	0.127	1.590
86	0.298	1000	1.7015	4.6324	0.286	3.173
84	0.338	200	6.0436	15.7919	0.398	1.608
85	0.342	500	2.8947	7.0354	0.323	3.919
93	0.542	500	1.8558	4.8893	0.520	6.522
92	0.556	200	4.0969	10.0872	0.575	4.633
95	0.769	500	1.3137	3.5670	0.741	10.842
94	0.788	200	2.9567	7.7682	0.815	6.522
91	1.092	100	4.0816	11.1451	1.194	5.404
90	1.100	200	2.1323	5.7986	1.141	10.182
81	1.476	100	3.0606	8.6304	1.602	7.391
83	1.485	500	0.6672	1.8611	1.467	34.329
82	1.568	200	1.4904	4.0959	1.637	19.350
80	1.564	50	5.5623	16.7528	1.781	1.013
89	1.678	200	1.4083	3.9463	1.739	18.509

3.4 Velocity measurement using square waves

Considering the results obtained in the above section, square waves were used as the heat inputs because the two phase differences which correspond to the sinusoidal heat inputs of different angular frequencies can be taken from the response of square wave. According to the results of the phase-difference method, the influence of apparent thermal diffusivity K can be eliminated by **Eq. 5.** Therefore, the square wave method is rather practical, for a precise sinusoidal heat input is difficult to induce and thermal diffusivity is difficult to ascertain. In these experiments, two phase differences in regard to the fundamental harmonic and the third higher harmonic were extracted from the response of square wave. **Photo 3** showns an example of the response of square wave. The experimental realuts are shown in **Table 2** and in **Fig. 2.** Velocities shown in **Fig. 2** are often observed in sand or loamy soils, which have large space between soil particles. The experimental results correspond well with the theoretical considerations. Therefore, the phase-difference method is proved to have sufficient accuracy for measuring directly the velocity of water in actual soil.

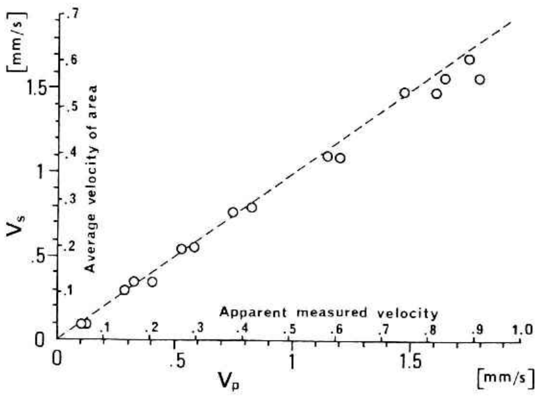

Fig. 2 Experimental results of water velocity measurement in porous media. Vp is obtained by the phase-difference method, and Vs is the total volume of flow divided by the sectional area of the experimental pipe. Calibrated velocity is shown outside the coordinates. The dashed line shows equal values of the calibrated Vp and the calibrated Vs.

4. Conclusion

Methods of velocity measurement using sinusoidal heat conduction are presented. The heat energy equation is shown, on which the two methods are developed. Next, the energy equation in porous media which include a flowing medium and a stationary medium is shown. According to the resultant equation, heat conduction in porous media is similar to that in simple fluids. Experiments in velocity measurement in porous media, made of the glass beads, were conducted by the phase-difference method. The velocity used in these experiments were 0.09 – 1.56 mm/s. The results show the practical validity of this method. The phase-difference method has some strong points for field use as follows:

(1) The soil is not contaminated chemically and the influences of thermal diffusivity can be eliminated by this method. Therefore, a long term and continuous measurement in the field can be performed because sampling the actual soil is unnecessary.

(2) The value of the temperature itself does not need to be measured. There is usually a time lag between the true temperature and the measured value, and thermal or electric noises disturb the measurement. In spite of these obstructions, the response is not influenced by high frequency noises and follows the varying thermal signal precisely, because the objective velocity is small in value and it needs to use a long period of input frequency. This is a strong point in field use.

(3) The soil-water velocity can be measured directly in the ground. It is common to use the coefficient of permeability to ascertain the movement of water in soil. However, the coefficient varies with the soil-water content, and the gradient of the total head in the soil is difficult to know. Generally, only one figure can be acceptable in use. Using the method presented, a greater accuracy can be attained and even the permeability can be evaluated.

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土中の水分移動量の測定

—多孔質物質中の流速測定実験—

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正弦波状温度入力を利用した多孔質物質中の真の流速の測定実験について述べている．最初に測定方法に関する既発表の理論（富永，1976 a，b)をまとめている．多孔質物質中での熱伝導方程式は純粋流体中での熱伝導方程式とよく似た形をしており，純粋流体中での正弦波状温度入力に対する応答の振幅および位相差に着目する２種の測定法がほとんどそのままの形で多孔質物質中の流速測定にも利用できる．次にガラス粒を用いて多孔質物質を構成し上記の理論の妥当性を検証している．一般に熱伝導系の１点に正弦波状温度入力を加えると空間的にも周波数が現われるので，まず入力周波数と空間的サンプリング間隔について考察を行なっている．さらに正弦波状入力を加えたときの応答が正しく正弦波になることから境界条件の妥当性を確認し，最後に方形波状温度入力に対する応答から多孔質物質中の流速測定を行っている。方形波入力に対する応答からはフーリェ解析により基本波と第３高周波に対する位相差を抽出できるので，位相差に着目する流速測定法によれば温度拡散率の影響を除去でき実際の応用に適している．実験での流速範囲は 0.09 − 1.56 mm／s で理論との良い一致を見た．

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